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| **Obuda University**<br>John von Neumann Faculty of Informatics | | | **Institute of Applied Mathematics** | |
| **Name and code:**<br>*Critical points and applications* | | | **Credits*:***<br>*2026/27 year I. semester* | |
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| Subject lecturers: Prof. dr. habil. Alexandru Kristály | | | | |
| Prerequisites (with code): | Calculus I, II | | | |
| Weekly hours: | Lecture: | Seminar.: | Lab. hours: | Consultation: |
| Way of assessment: | | | | |

**Course description:**

*Goal*: To provide an insight into the theory of critical points and its applications.

*Course description:* Several mathematical problems can be reduced to the study of critical points of a certain energy functional. During the course we provide a quite complete picture about this theory, showing both theoretical aspects and applications in partial differential equations (PDEs) and differential geometry.

**Lecture schedule**

| *Education week* | *Topic* |
|---|---|
| 1. | Motivation to study critical points |
| 2. | Deformation lemmas |
| 3. | Morse theoretical approach to critical points |
| 4. | Mountain pass theorem and Palais-Smale condition |
| 5. | Application: elliptic PDEs on bounded domains |
| 6. | Principle of symmetric criticality and Strauss lemma |
| 7. | Rubic actions and elliptic PDEs on unbounded domains |
| 8. | Nonsmooth critical points and differential inclusions |
| 9. | Variational inequalities |
| 10. | Application: subcritical PDEs |
| 11. | Application: sublinear PDEs |
| 12. | Application: elliptic PDEs with oscillations |
| 13. | Critical points on non-flat structures |
| 14. | Open problems and perspectives in critical points |

**Midterm requirements**

| *Education week* | *Topic* |
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**Final grade calculation methods**

| Achieved result | Grade |
|---|---|
| 89%-100% | excellent (5) |
| 76%-88<% | good (4) |
| 63%-75<% | average (3) |
| 51%-62<% | satisfactory (2) |
| 0%-50<% | failed (1) |

2026. 06. 10.

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| **Type of exam** |
| Project presentation & Written exam |
| **Type of replacement** |
| Project presentation |
| **References** |
| Mandatory:<br>1. Costea N, Kristály A, Varga C, *Variational and Monotonicity Methods in Nonsmooth Analysis*, Frontiers in Mathematics, Birkhäuser/Springer, 2021.<br>2. Kristály A., Radulescu V., Varga Cs., *Variational Principles in Mathematical Physics, Geometry, and Economics*, Cambridge University Press, Enciclopedia of Mathematics and its Applications. No 136, 2010.<br>3. Willem M., *Minimax theorems*. Progress in Nonlinear Differential Equations and their Applications, 24. Birkhäuser Boston, Inc., Boston, MA, 1996. |
| Recommended:<br>1. Gasiński L., Papageorgiou N., *Nonsmooth critical point theory and nonlinear boundary value problems.* Series in Mathematical Analysis and Applications, 8. Chapman & Hall/CRC, Boca Raton*, FL,* 2005.<br>2. Kristály A., Moroşanu Gh., New competition phenomena in Dirichlet problems. *J. Math. Pures Appl*. (9) 94 (2010), no. 6, 555–570.<br>3. Kristály A., Infinitely many solutions for a differential inclusion problem in R*N*. *J. Differential Equations* 220 (2006), no. 2, 511–530. |